

# Modern topics in theoretical nuclear physics

B.K. Jennings<sup>1,\*</sup> and A. Schwenk<sup>1,2,†</sup>

<sup>1</sup>*TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, Canada, V6T 2A3*

<sup>2</sup>*Nuclear Theory Center, Indiana University, Bloomington, IN 47408*

prepared for the Nuclear Physics Town Hall Meeting at TRIUMF, Sept. 9-10, 2005

Over the past five years there have been profound advances in nuclear physics based on effective field theory and the renormalization group. In this brief, we summarize these advances and discuss how they impact our understanding of nuclear systems and experiments that seek to unravel their unknowns. We discuss future opportunities and focus on modern topics in low-energy nuclear physics, with special attention to the strong connections to many-body atomic and condensed matter physics, as well as to astrophysics. This makes it an exciting era for nuclear physics.

## I. INTRODUCTION

Nuclear physics is undergoing a renaissance. This results from a confluence of novel interests, experimental opportunities and significant developments in theoretical nuclear physics.

First, there is a need for precise nuclear input to astrophysical calculations. Much that happens in the sky has a nuclear physics underpinning, from how the sun shines to what happens when stars explode in a supernova. The formation of the elements, nucleosynthesis, requires the knowledge of nuclear masses, lifetimes and reactions at low energies. Astrophysical interests tied with the quest for an understanding of nuclear systems at the limits of stability have motivated the construction of radioactive beam facilities worldwide: ISOLDE at CERN, SPIRAL at GANIL, NSCL at MSU, ISAC at TRIUMF, RIBF at RIKEN (under construction), FAIR at GSI (construction begin 2008), RIA (planned with highest construction priority) and EURISOL (proposed). With these experimental facilities, it is now possible to vary the composition of known nuclei to extreme neutron to proton ratios and investigate their phenomena. Nuclear physics also uses many new experimental techniques, such as ion or atom traps for high-precision studies. In addition, nuclei play an important role in constraining the standard model of particle physics, for example the  $V_{ud}$  parameter in the CKM matrix and the Dirac or Majorana nature of neutrinos.

From a condensed matter perspective, studies of rare isotopes may be regarded as “femtoscience” research, where one starts with known nuclei and studies their dependence on “doping” by neutrons or protons. As in condensed matter physics, some key aspects of novel nuclear systems are superfluidity/superconductivity and the phenomena of frustrated systems, for nuclei due to the competition of the long-range electromagnetic and short-range strong interactions.

It is readily anticipated that the frontiers investigated with radioactive beam facilities will lead to industrial and medical applications, similar to the techniques that stem from prior university-based accelerator facilities. Today, one of three hospitalized patients undergoes a nuclear medicine procedure.

Astrophysics questions and exciting nuclear experiments are spurring a new generation of nuclear theorists to develop systematic approaches to the nuclear many-body problem. In particular, a novel understanding of nuclear interactions based on effective field theory and the renormalization group plays a crucial role in the theoretical advances.

In this brief, we identify three main aspects of current research in nuclear physics. The first is as an extremely rich and complex nuclear many-body problem that spans 18 orders of magnitude from nucleons to neutron stars and is ultimately based on QCD. It is therefore important to develop a reliable and coherent description of nuclear phenomena over this range. Since the many-body methods used for nuclei and neutron stars are general, theoretical progress in nuclear physics can have direct impact for instance on the theory of quantum dots and cold atoms. The development of many-body methods reaches across science: The coupled cluster technique was first proposed in the context of nuclear physics, further developed with many extensions in quantum chemistry, and now has returned to nuclear physics for use in ab-initio calculations of the properties of medium-mass nuclei. Similarly, renormalization group methods that were proposed for electronic systems, such as the Hubbard model, have recently been applied to nucleonic matter and superfluidity in neutron stars.

Second, nuclear physics is an essential part of astrophysics. There would have been no solar neutrino problem, if the nuclear physics and hydrodynamics of the sun had not been sufficiently well understood to make accurate predictions of the expected neutrino flux. Many other astrophysical processes intimately involve nuclear physics. The equation of state and the neutrino response of nuclear matter are crucial inputs to simulations of supernova explosions and neutron star cooling. Nuclear masses, lifetimes and reactions drive nucleosyn-

---

\*E-mail: jennings@triumf.ca

†E-mail: schwenk@triumf.ca

thesis. Nuclear energy and gravity are the prime movers in the universe.

The third aspect of nuclear physics is as a necessary part of experiments needed for particle physics or other subfields of physics. For example, the parameter  $V_{ud}$  in the CKM matrix can be determined from nuclear  $\beta$  decay. Here the nucleus can be considered as part of the target holder of the quark undergoing  $\beta$  decay, and clearly a successful experiment requires that the relevant nuclear physics be well understood.

The second and third aspects can obviously only take place in the context of the first. The role of the nucleus in the cosmos cannot be understood without first understanding the nucleus in the laboratory.

In this brief, we first discuss how effective field theory and the renormalization group place nuclear interactions in the hierarchy of theories that range from atomic and condensed matter to high-energy particle physics. These ideas lead to a change in our understanding of the nuclear many-body problem and to the development of novel and refined methods, which put nuclear many-body theory in the main stream of general many-body physics. The advances also have important consequences for experiments. Throughout this presentation, we focus on modern topics in low-energy nuclear physics, with special attention to the various strong connections to atomic, condensed matter and astrophysics.

## II. THE BIG PICTURE

The reductionist goal is to reduce everything to one all-encompassing theory. At the Planck scale  $M_P \sim 10^{19}$  GeV, string theory holds that promise. The verification of string theory is complicated however, since it demands either extremely high-energy probes or reliable calculations of very small contributions to low-energy processes. Between the Planck scale and the electro-weak breaking scale,  $M_{ew} \sim 10^2$  GeV, little is known. In this energy range there are a number of possible effective field theories, such as grand unified theories, supersymmetric theories or left-right symmetric models.

At the electro-weak breaking scale  $M_{ew}$ , the standard model of strong and electro-weak interactions is well explored and extraordinarily successful. (The dynamic mechanism of electro-weak symmetry breaking and the nature of the Higgs boson are still open problems.) However, it is certainly not the “theory of everything”, since it does not describe gravity. Therefore the question arises: How can a theory be successful, when the underlying theory to which it is an approximation is unknown? This is where the idea of renormalization comes in. The physics at distances too short to be probed is not resolved and can be replaced by simpler interactions whose couplings encode all short-distance contributions. This leads to an effective field theory with a finite number of low-energy constants. The masses of the elementary particles are such constants. They can either be determined exper-

imentally, or by matching amplitudes calculated in the effective field theory and the underlying theory. While details of the underlying theory are not needed to make predictions at low energies, the higher energy theory may unify the phenomena of the standard model and reduce the number of its parameters.

At the next energy scale, we have QCD as the theory of strong interactions. QCD can be divided into three regimes. At high energies,  $M_{pQCD} \gtrsim \text{few GeV}$ , QCD becomes asymptotically free and interactions of quarks and gluons are perturbative. At intermediate energies,  $M_{had} \sim 1$  GeV, the physics is that of strongly-interacting hadrons, with a diverse spectrum of mesons and baryons and their resonances. Nuclear physics governs the interactions of the lightest baryons and the lightest mesons at energies  $M_{nuc} \sim 100$  MeV. The nuclear physics regime naturally carries a key imprint of QCD, since the pions, the lightest mesons, are the Goldstone bosons of chiral symmetry breaking. Each of the three regimes have their own relevant degrees of freedom and thus their own effective theory. Effective field theory allows us to separate nuclear physics from the more complicated problem of hadronic physics, while maintaining a clear and direct connection to QCD. At lower energies, we have atomic and condensed matter physics, where nuclear and standard model parameters (nuclear masses, charges, charge radii) are low-energy constants. The resulting picture of physics thus consists of layer upon layer of effective field theories, and all interactions are effective interactions in this sense.

The problem at every energy scale decouples from the physics at higher energies. The high-energy information is encoded in the low-energy coefficients of the effective field theory. This decoupling is very useful, even when the effective theory at the higher energies is known, as with QCD and nuclear physics. This is because the effective field theory focuses on the physics of the problem at hand, and provides a method to separate out the relevant information from complicated details that are not needed. For example, Fermi’s theory of  $\beta$  decay works well without explicitly including the  $W$  and  $Z$  bosons.

The idea of effective field theory and decoupling run counter to the reductionist’s ideal. The physics at every energy scale can be studied largely independent of what happens at other energies. It is not necessary to understand interactions at higher energies to make progress at a given scale. An extreme example is critical phenomena, where the critical exponents depend only on very general properties, the symmetries and the dimensionality, of the bare Lagrangian. All the remaining information is integrated out, and the results depend only on the fixed points of the renormalization group. It is the renormalization group and not the bare couplings that determine the critical exponents.

As discussed in the next section, a similar discovery was recently made in nuclear physics: Microscopic approaches to nuclear systems traditionally start from various models for the inter-nucleon interaction, which differ

substantially at short distances and lead to model dependences for nuclei and neutron stars. Using the renormalization group, it was shown that all these microscopic nuclear forces evolve to a universal nucleon-nucleon interaction for nucleons with momenta  $k \lesssim 4M_{\text{nuc}}$ , when the model-dependent high-momentum parts are integrated out. This unifies nuclear structure and nuclear astrophysics calculations. Nuclear interactions can be thought of as one in the sequence of effective interactions that stretch from string theory at highest energies through the standard model of particle physics to the effective interactions of atomic and condensed matter physics.

### III. NUCLEAR INTERACTIONS BASED ON EFT AND RG

The goal of nuclear theorists is to provide a coherent and systematic description of all low-energy nuclear phenomena, those occurring on earth and in stars. The theoretical framework can be used to predict nuclear processes that are currently not accessible experimentally, and to understand the microphysics of observed phenomena. Therefore, nuclei, neutron stars and supernovae serve as laboratories to test effective interactions at nuclear energies. A rigorous test requires that the truncations in the nuclear Lagrangian are well understood and that the microscopic many-body calculations are reliable. In this section, we review the advances in our understanding of nuclear interactions based on Effective Field Theory (EFT) and the Renormalization Group (RG). We will discuss recent progress in many-body calculations in the following section.

The connection of nuclear interactions to QCD is through EFT. When nuclear systems are probed at low energies, the relevant degrees of freedom are nucleons and pions, the lightest confined baryons and mesons respectively. Nuclear EFT includes explicitly nucleon and pion fields and allows all possible interactions that are consistent with the symmetries of QCD, most importantly chiral symmetry. In order to be predictive and systematic, an organization (“power counting”) must be present to permit a finite truncation of possible terms in the Lagrangian. For nuclear interactions, the power counting originally proposed by Weinberg [1] is in powers of the typical momenta in nuclear systems  $Q$  over the EFT breakdown scale  $\Lambda_\chi$ . An estimate for the low-momentum scale is  $Q \sim m_\pi$ , where  $m_\pi \approx 140$  MeV is the pion mass, and the EFT breakdown scale is  $\Lambda_\chi \sim 700$  MeV, where heavier mesons (like the  $\rho$  meson with  $m_\rho \approx 770$  MeV) and nucleon resonances are resolved and have to be included explicitly. The pions are the Goldstone bosons of chiral symmetry breaking, and consequently their interactions become weaker at low energies. This is a key QCD property, which is clearly present in its EFT. The low-energy EFT does not include explicitly heavier baryons or quarks and gluons. All their effects are present in the low-energy couplings of the simpler short-

range interactions. For further details we refer the reader to several excellent reviews on EFT applied to nuclear forces [2, 3] and also to the general introduction to EFT by Lepage [4].

Nuclear interactions based on EFT have now been developed to Next-to-Next-to-Next-to Leading Order,  $N^3\text{LO}$  or up to  $(Q/\Lambda_\chi)^4$ , in the low-momentum expansion [5, 6]. To this order, one has 24 low-energy constants, which have been fit to the world set of  $\sim 4500$  neutron-proton and proton-proton scattering data. A substantial number of these data have been measured at TRIUMF. The reproduction of two-body observables is very good with  $\chi^2/\text{datum} \approx 1$ . We emphasize that the fit to experiment automatically takes into account all high-energy effects on low-energy observables. Alternatively, since nuclear EFT is connected to QCD, the low-energy constants can be determined using lattice QCD or other non-perturbative methods. The continuous advances in lattice QCD, for example full simulations with light quarks, make this a long-term vision for nuclear forces. For exciting progress on nuclear physics from QCD see [7].

The physics of nuclear EFT is extremely rich. The pionic interactions lead to significant non-central parts in nuclear forces. These are analogous to magnetic dipole-dipole interactions and depend on the orientation of the nucleon spins relative to their position. In many-body systems, such interactions can lead to remarkable phenomena, for instance the differences observed in the A and B phases of liquid  $^3\text{He}$  recognized with the 2003 Physics Nobel Prize [8]. The spin-independent part of nuclear interactions is similar to molecular van-der-Waals potentials, and thus nuclear systems combine the phenomena of spin systems and atomic systems.

There are several key advantages of nuclear EFT. In nuclear physics, many-body forces are inevitable. (Note that QCD has a three-quark interaction through the non-Abelian  $ggg$  vertex.) It is established beyond doubt that for all realistic nucleon-nucleon (NN) interactions, a significant three-body force is required to describe light nuclei [9, 10, 11, 12]. In nuclear EFT, the power counting predicts that the first three-nucleon (3N) interaction is present at  $N^2\text{LO}$ ; their effects are thus suppressed by  $(Q/\Lambda_\chi)^3$  compared to NN interactions [13, 14]. A similar hierarchy can be derived for many-body forces using nuclear EFT. The EFT many-body forces are consistent with the NN interaction, for example the long-range part of the 3N and 4N forces are parameter-free; their couplings also enter the NN interaction and are constrained from  $\pi N$  or  $\pi\pi$  scattering respectively. Moreover, the  $N^3\text{LO}$  3N force only has two low-energy constants.

In addition to a controlled and viable expansion of many-body forces, the nuclear EFT approach can address questions like: How would nuclear shell structure or nucleosynthesis change, if the up and down quark masses would be different? A first investigation of this sort was the quark-mass dependence of the binding energy of the deuteron [15, 16], which is the lightest nucleus and important to nuclear burning. Moreover, neutrino-nucleon [17]

or parity-violating interactions [18] can be consistently incorporated in applications of nuclear EFT to studies of fundamental symmetries.

The second advancement to our understanding of nuclear interactions comes directly from the RG. The idea behind renormalization is that the effects of the high-momentum modes on the low-momentum theory can be simulated by a set of simpler interactions. In the regime of the EFT expansion, the simpler interactions are contact interactions and their derivatives. Using the RG, it is possible to integrate out modes with momenta larger than a cutoff  $\Lambda$  starting from NN interactions. The resulting effective interaction, defined for momenta below  $\Lambda$ , encodes all high-energy effects in the low-energy couplings. While nuclear physics may seem unfamiliar with the renormalization terminology, the basic ideas of the RG are present in the seminal work of Bloch and Horowitz [19] and in subsequent nuclear many-body methods.

To illustrate the power of the RG in nuclear physics, we review the application to nuclear interactions [20, 21]. RG invariance requires that the low-momentum theory reproduces the same low-momentum scattering amplitude  $T$  as a given large cutoff nuclear interaction with high-momentum modes. Therefore, the low-momentum interaction  $V_{\text{low } k}$  satisfies

$$T(\mathbf{k}', \mathbf{k}, E_{\mathbf{k}}) = V_{\text{low } k}^{\Lambda}(\mathbf{k}', \mathbf{k}) + \int^{\Lambda} \frac{d^3 \mathbf{p}}{(2\pi)^3} V_{\text{low } k}^{\Lambda}(\mathbf{k}', \mathbf{p}) G^{(2)}(\mathbf{p}, E_{\mathbf{k}}) T(\mathbf{p}, \mathbf{k}, E_{\mathbf{k}}), \quad (1)$$

where  $\mathbf{k}'$  and  $\mathbf{k}$  are the incoming and outgoing momenta, the scattering energy is  $E_{\mathbf{k}}$ , and  $G^{(2)}(\mathbf{p}, E_{\mathbf{k}})$  denotes the two-particle propagator. The cutoff in Eq. (1) defines the resolution scale of the effective theory, since details at distances  $r \lesssim 1/\Lambda$  are not resolved. The cutoff independence of the scattering amplitude,  $dT(\mathbf{k}', \mathbf{k}, E_{\mathbf{k}})/d\Lambda = 0$ , leads to a RG equation that determines how  $V_{\text{low } k}$  evolves as the cutoff is lowered and high-momentum modes are integrated out.

In the left part of Fig. 1, we show the  $N^3\text{LO}$  EFT, as well as various models for the NN interaction. These potential models all include the same long-range one-pion exchange interaction and are fit to scattering data for momenta  $k \lesssim 2.1 \text{ fm}^{-1}$  (roughly up to the threshold for pion production), but they differ substantially in their treatment of the high-momentum physics. When these model dependences are removed using the RG, one obtains a universal low-momentum interaction  $V_{\text{low } k}$  for all cutoffs  $\Lambda \lesssim 2.1 \text{ fm}^{-1}$ . This is shown in the right part of Fig. 1. The RG thus unifies microscopic nuclear forces and removes these model dependences from many-body observables.

For EFT interactions, the RG can be used to change the resolution scale in nuclear forces, analogous to evolving the renormalization scale in parton distribution functions in QCD. The RG evolution of EFT interactions to lower cutoffs generates all higher-order short-range inter-

actions necessary to maintain RG invariance, and therefore it has been argued that  $V_{\text{low } k}$  effectively parameterizes a higher-order nuclear EFT interaction [22]. In summary, instead of a number of models, we now have inter-nucleon interactions (from EFT and RG) that depend on the resolution scale. This is similar to the running coupling in QCD and to improved lattice actions, where the resolution is  $\Lambda = \pi/a$ ,  $a$  being the lattice spacing.

For cutoffs below the breakdown scale, the scaling of higher-order two-body or many-body forces is dominated by the cutoff, because  $Q/\Lambda > Q/\Lambda_{\chi}$ . Many-body observables that do not include many-body forces will be cutoff dependent and the cutoff variation gives an estimate for the neglected physics. Varying the cutoff in nuclear interactions is thus a powerful tool to assess the truncation errors and the completeness of the calculation. Such theoretical uncertainties are clearly important for predictions of nuclear systems, for terra incognita at ISAC or in stars. They also provide valuable guidance, where future experimental constraints are most needed.

The RG casts very useful insights into why the different potential models have worked as well as they have and shows their limitations. One criticism of the traditional potential models is that their short-range parts should be constructed from explicit quark degrees of freedom. They should not be simple interactions, often taken to be local or approximately local, based on heavy-meson exchanges, dispersion theory or in some cases phenomenology. The EFT and RG understanding, however, shows that it is entirely legitimate to replace physics that is not resolved by something simpler [4]. The higher energy physics, including quark effects, is taken into account by fitting these simpler interactions to experiment. The problem with the potential models is not that the short-distance details are incorrect, but more so that they lack a systematic scheme to construct consistent many-body interactions and effective operators. In addition, the potential models have strong high-momentum modes and large cutoffs, and one could be misled into thinking a higher cutoff implies that the physics is more valid. While the RG approach shows that the different potential models lead to the same low-momentum theory, it also demonstrates that their high-momentum behavior is not constrained by data. Theoretical studies that are sensitive to these short-range parts are at best incomplete.

An important result is that 3N interactions become weaker for lower cutoffs [22]. This shows that nuclear interactions with large cutoffs are counterproductive, because 3N forces in this case must cancel loop contributions from high-momentum intermediate states that are incorrectly represented. 3N interactions corresponding to  $V_{\text{low } k}$  are perturbative in light nuclei [22] and therefore tractable. As a result, we will be able to perform the first calculations with 3N forces for intermediate-mass nuclei [23]. Fundamental progress in nuclear physics requires a better understanding of 3N interactions. These lead to particular density and isospin dependences, and experimental information from nuclei therefore provides

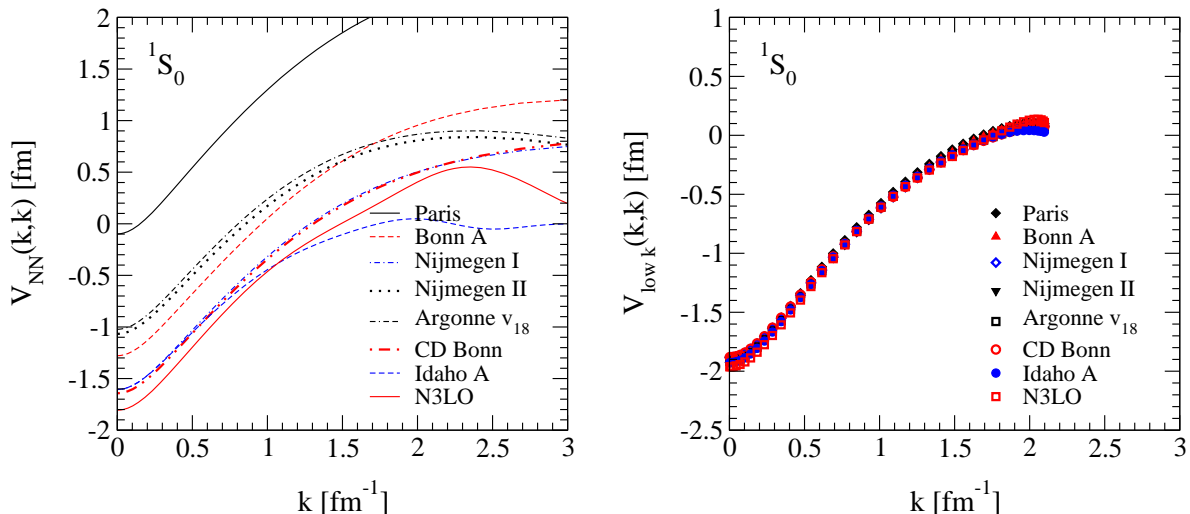


FIG. 1: Left figure: Different nuclear interaction models  $V_{NN}(k,k)$  versus incoming/outgoing relative momentum  $k$ . Right figure: The resulting low-momentum interactions  $V_{\text{low } k}(k,k)$ , derived by integrating out the high-momentum ( $p > 2.1 \text{ fm}^{-1}$ ) modes using the RG. The interactions are shown for the  $^1S_0$  partial wave, but the same universal behavior is found in all partial waves and for lower cutoffs [20, 21]. The momentum units are  $1 \text{ fm}^{-1} \approx 200 \text{ MeV}$ .

significant constraints. The ISAC facility can play a unique role in this endeavor.

A weakening of the 3N force is also observed empirically in the Unitary Correlation Operator Method [24], where an explicit unitary basis transformation is developed that removes the short-range correlations. The resulting effective interaction is similar to  $V_{\text{low } k}$ , and as in the RG approach, this implies that short-range correlations are not strict observables. In addition, large-basis diagonalizations within the No-Core Shell Model show a similar behavior. As the basis includes more high-energy states of the NN interaction, one finds a deterioration from the experimental binding energies [25].

Effective interactions have a long history in nuclear physics. In the Bloch-Feshbach-Horowitz projection operator formalism [19, 26] (the same framework is used to understand and predict Feshbach resonances in cold atoms), the effective interaction carries a characteristic energy dependence. For many-body systems, a correct treatment of this energy dependence is complicated, with promising progress for light nuclei [27]. The RG approach of Refs. [20, 21] and equivalent unitary transformations [28] lead to energy-independent interactions. Thus the concept of effective interactions is more general than sometimes realized [29].

In addition to the interplay between NN and 3N interactions, there are effective operators that are also resolution scale dependent and evolve with cutoff. To see why effective operators are necessary, consider for example electromagnetic interactions. Photons can couple to quarks inside nucleons and to heavier mesons/baryons that have been integrated out. Electromagnetic operators must include these effects. Nuclear EFT provides a systematic scheme to incorporate all contributions with simpler interactions. We stress that effective operators

and many-body forces cannot be meaningfully discussed outside the context of a given two-body interaction.

We conclude this section by discussing nuclear interactions at very low energies, where even the pion is not resolved. The corresponding EFT is constructed from contact interactions and their derivatives. Pionless EFT has been extremely successful for strongly-interacting systems with large scattering lengths [30]. These systems exhibit universal properties at low densities, independent of the atomic or nuclear details, because there are no lengths scales associated with the interaction in this regime. For example, for two spin states with equal populations, the equation of state of cold gases of  $^6\text{Li}$  or  $^{40}\text{K}$  atoms in the vicinity of Feshbach resonances is identical to the equation of state of low-density neutron matter in the crust of neutron stars. Applications of the pionless EFT developed in nuclear physics range from cold atoms [31] to nuclear reactions [32] and neutron stars [33]. There are many nuclear reactions with large scattering lengths and thus progress in their theoretical description can be directly applied to describe cold atom gases with similar resonant interactions.

Finally, the EFT and RG again provide useful insights why simple nuclear potential models developed for nuclear reactions work well. For example, the recent calculation of the  $^7\text{Be}(p,\gamma)^8\text{B}$  reaction with simple, pionless interactions is rather successful [34]. In the new understanding, these simpler interactions are nuclear interactions at a lower resolution scale, which can be thought of as integrating out the pions in nuclear interactions. The EFT and RG thus provide a way to build on the empirical successes of these calculations, in a way that other modeling does not.

#### IV. THE NUCLEAR MANY-BODY PROBLEM BASED ON EFT AND RG

The traditional problem in nuclear physics is that nuclear interactions are strong and model dependent (see e.g., the discussion in [35]). Both aspects can be traced to the same sources. First, different models for the strongly repulsive core lead to different behavior at high momenta or high virtual energies. Second, nuclear interactions have tensor forces that scatter strongly to high momenta. The optimal way to deal with the high-momentum modes is to convert the problem first to a low-momentum theory more appropriate to the resolution at hand. This realization has led to a revolution in nuclear physics. A common feature in modern nuclear many-body developments is that they first integrate out (and thus suppress) high-momentum virtual contributions [21, 24, 27, 36, 37]. In the process, low-momentum effective interactions are generated. This is achieved either directly by the RG [20, 21] or through other effective interaction methods [24, 27, 36, 37] (for comparative details see [38]). This separation of the long-range physics from the short-range details builds on the separation of scales in the hadron spectrum of QCD.

The treatment of high-momentum modes has been a continual difficulty in nuclear physics. The success of Dirac phenomenology [39] has been traced to the fact that it suppresses high loop momenta [40, 41]. In pion-nucleus scattering there is a similar improvement when high-momentum loop contributions are suppressed [42]. As discussed in the previous section, nuclear interactions with lower cutoffs or those that suppress high-momentum modes have weaker 3N forces. It is intriguing that these effects correlate with a weaker iterated pion exchange, and investigations in this direction may lead to valuable insights.

A significant advantage of low-momentum interactions is that they can be directly applied to nuclear many-body systems with model-independent results and without uncontrolled resummations [35]. For systems with  $A < 100$  particles, the prime approaches are the nuclear shell model [43] and the coupled cluster method [37]. Presently, exact shell-model diagonalizations are possible for all semi-magic nuclei, and for  $A < 70$  nuclei in  $0\hbar\omega$  space. The coupled cluster method is the method of choice for systems with up to 100 electrons in quantum chemistry. First applications of low-momentum interactions to the nuclear shell model are very promising [44]. These studies can provide a basis for future calculations of the exotic structure of nuclei investigated at ISAC. As discussed, 3N interactions become weaker for low-momentum cutoffs and are thus tractable in larger systems. As a result, we will be able to include microscopic 3N interactions beyond the lightest nuclei. The first coupled cluster calculations with this endeavor are under way [23].

For bulk properties of nuclear matter, low-momentum interactions offer the possibility of a perturbative and

therefore systematic approach [35]. In contrast to the traditional perception, the role of the 3N interaction is essential for saturation. These results demonstrate that the power counting must change for nuclear matter [35], and the cutoff scaling of different contributions can provide guidance for the development of a systematic EFT. In addition, the nuclear matter results imply that exchange correlations are tractable, and this motivates a program to derive the nuclear density functional from microscopic interactions. Density Functional Theory (DFT) is the method of choice to study ground state properties of  $A > 100$  particle systems [45, 46, 47, 48]. The density functional is universal and the microscopic foundations of DFT are well-understood. In a path integral approach, the density functional is the effective action for the density. We refer the reader to [49] for a short discussion of the microscopic foundations and DFT opportunities in the context of nuclear physics. Here, we only remark that the nuclear density functional comes with novel and unique features due to superfluidity and the self-bound nature of nuclei. These are largely unexplored for electronic systems.

In addition, there are promising non-perturbative RG methods for nuclear many-body systems [50], based on the RG approach to interacting Fermi systems proposed by Shankar [51]. For condensed matter systems, this approach has been applied to study the phase diagram of the two-dimensional Hubbard model [52, 53, 54] and electronic excitations in atoms [55]. In nuclear physics, the RG method was first used to investigate superfluidity in neutron stars [50], since pairing properties are a pivotal input to simulations of neutron star cooling. A systematic extension of the RG method to finite systems will lead to non-perturbative shell-model interactions and effective operators for heavy nuclei.

The long-term vision of nuclear many-body theory is a microscopic and predictive understanding of nucleonic matter under extreme compositions, temperatures and densities, with theoretical error estimates. There are many common themes with atomic and condensed matter systems: How does the structure of matter change with its composition? What is the many-body physics of complex and collective phenomena in nuclear systems? How are shape transitions in nuclei and neutron stars related to frustrated systems and the phase diagram of asymmetric matter? What are the connections to mesoscopic systems and chaos? In addition, the EFT connection to QCD makes it possible to address how nuclear many-body phenomena depend on QCD parameters, for example the quark masses.

Reliable many-body approaches are especially important for the nuclear physics input to stars, supernovae and the formation of the elements. The nuclear equation of state is key to supernova and neutron stars, where nucleonic matter range from a complex gas at extremely low densities to a liquid at nuclear densities. Below saturation density, nucleonic matter comes in various shapes known as “nuclear pasta”. Experiments with low-energy

heavy-ion collisions and connections to frustrated systems can provide insights on the cluster physics of the complex gas and the nuclear pasta. Recent progress comes from molecular dynamics simulations of the pasta phases [56, 57] and a model-independent description of low-density nuclear matter based on the virial expansion [58]. The resulting virial equation of state constrains the physics of the neutrinosphere in supernovae, and differs from all models used in supernova simulations. In addition, neutrino interactions with nucleonic matter are key to supernova explosions and the cooling of neutron stars. The many-body advances make it possible to derive reliable neutrino-matter interactions consistent with the nuclear equation of state and including the effects due to nuclear clusters. Moreover, nuclear superfluidity strongly modifies the cooling of neutron stars, since pairing of neutrons or protons suppresses neutrino emission via  $\beta$  decay and neutrino bremsstrahlung. It is exciting that there is a similar observation in nuclei: For the halo nucleus  $^{11}\text{Li}$ , the  $\beta$  decay occurs in the  $^9\text{Li}$  core, since the pairing energy suppresses the decay of the two halo neutrons [59].

Since the nucleon-nucleon scattering lengths are large, the nuclear many-body problem has an exciting overlap with the universal physics of cold atoms with resonant interactions. The physics of low-density neutron matter is part of the BEC-BCS crossover physics studied with trapped Fermi gases [60, 61, 62, 63, 64]. Likewise, many nuclear reactions have large scattering lengths, and a model-independent description of nuclear reactions can therefore be used to study other resonant interactions. Another exciting overlap exists with asymmetric [65] or rotating Fermi gases and asymmetric nuclear matter, rotating nuclei or neutron stars.

## V. CONCLUSIONS

Nuclear physics covers the low-energy regime of QCD and explores the interactions of nucleons in nuclei and astrophysical systems. Nuclear interactions are effective interactions for the relevant energies, and present one rung in the ladder of effective theories. Nuclear systems should therefore not be thought of as a simple extension of hadronic physics to lower energies, but rather as

a separate many-body problem with distinct degrees of freedom. The different effective theories should not be thought of as more or less fundamental. What drives the physics at one level is frequently not the details of the next level, but rather the general aspects of symmetries, renormalization and many-body physics. What is important in understanding nuclear physics in the laboratory and in stars is not the EFT at some higher energy scale, but rather the effective interaction at the energies of the phenomena of interest.

Nuclear interactions have been derived in an EFT for low-energy QCD. They depend on the cutoff or the resolution scale of the effective theory and the renormalization is identical to the running couplings in quantum field theories. The RG can be used to change the resolution scale in nuclear interactions, and we have shown how all microscopic nuclear forces evolve to a universal NN interaction for nucleons with momenta  $k \lesssim 2.1 \text{ fm}^{-1}$ . In contrast to the different potential models used traditionally in nuclear physics, we now have a low-momentum interaction that depends on the resolution scale. In addition, nuclear EFT offers for the first time a consistent and practical expansion scheme for many-body interactions and the coupling to electromagnetic/weak probes. As a result, calculations with microscopic 3N forces beyond the lightest nuclei are now possible.

New frontiers in nuclear physics will be set by model-independent predictions of nuclear structure and nucleonic matter. The nuclear many-body problem shares many of the approaches and methods with atomic, condensed matter and high-energy many-body problems. All modern advances in nuclear many-body physics reach over physics subfield barriers: for example, EFT for few-nucleon systems and cold atoms, the Coupled Cluster method and DFT for nuclear and electronic structure, and the RG for superfluidity in neutron stars and low-dimensional Fermi systems. While nuclear physics exhibits all features of many-body systems, it is distinguished by unique aspects: for example, the connection to QCD, its importance to astrophysical systems and to our very own existence, and its key part to the studies of fundamental symmetries. Nuclear physics therefore has and will contribute strongly to the general understanding of many-body systems.

- 
- [1] S. Weinberg, Phys. Lett. **B251** (1990) 288; Nucl. Phys. **B363** (1991) 3.
  - [2] U. van Kolck, Prog. Part. Nucl. Phys. **43** (1999) 337.
  - [3] S.R. Beane, P.F. Bedaque, W.C. Haxton, D.R. Phillips and M.J. Savage, At the Frontier of Particle Physics, Ed. M. Shifman, Vol. 1, p. 133, World Scientific, nucl-th/0008064.
  - [4] G.P. Lepage, “How to Renormalize the Schrödinger Equation”, Lectures given at 9th Jorge Andre Swieca Summer School: Particles and Fields, Sao Paulo, Brazil, February, 1997, nucl-th/9706029.
  - [5] D.R. Entem and R. Machleidt, Phys. Rev. **C68** (2003) 041001(R).
  - [6] E. Epelbaum, W. Glöckle and U.G. Meißner, Nucl. Phys. **A747** (2005) 362.
  - [7] M.J. Savage, PoS Lattice2005, hep-lat/0509048 and talk at PANIC05 online at <http://www.panic05.lanl.gov>.
  - [8] A.J. Leggett, 2003 Physics Nobel Lecture, Rev. Mod. Phys. **76** (2004) 999.
  - [9] S.C. Pieper and R.B. Wiringa, Ann. Rev. Nucl. Part. Sci. **51** (2001) 53.
  - [10] S.C. Pieper, R.B. Wiringa and J. Carlson, Phys. Rev.

- C70** (2004) 054325.
- [11] A. Nogga, H. Kamada and W. Glöckle, Phys. Rev. Lett. **85** (2000) 944.
  - [12] P. Navratil and W.E. Ormand, Phys. Rev. **C68** (2003) 034305.
  - [13] U. van Kolck, Phys. Rev. **C49** (1999) 2932.
  - [14] E. Epelbaum, A. Nogga, W. Glöckle, H. Kamada, U.G. Meißner and H. Witala, Phys. Rev. **C66** (2002) 064001.
  - [15] S.R. Beane and M.J. Savage, Nucl. Phys. **A717** (2003) 91 (2003); **A713** (2003) 148.
  - [16] E. Epelbaum, U.-G. Meißner, and W. Glöckle, Nucl. Phys. **A714** (2003) 535.
  - [17] M. Butler, J.-W. Chen and X. Kong, Phys. Rev. **C63** (2001) 035501; K. Kubodera and T.S. Park, Ann. Rev. Nucl. Part. Sci. **54** (2004) 19.
  - [18] S.L. Zhu, C.M. Maekawa, B.R. Holstein, M.J. Ramsey-Musolf and U. van Kolck, Nucl. Phys. **A748** (2005) 435.
  - [19] C. Bloch, Nucl. Phys. **6** (1958) 329; C. Bloch and J. Horowitz, Nucl. Phys. **8** (1958) 91.
  - [20] S.K. Bogner, T.T.S. Kuo, A. Schwenk, D.R. Entem and R. Machleidt, Phys. Lett. **B576** (2003) 265.
  - [21] S.K. Bogner, T.T.S. Kuo and A. Schwenk, Phys. Rept. **386** (2003) 1.
  - [22] A. Nogga, S.K. Bogner and A. Schwenk, Phys. Rev. **C70** (2004) 061002(R).
  - [23] Coupled-cluster calculations with the low-momentum NN and 3N interaction are in progress; D.J. Dean *et al.*, in preparation.
  - [24] R. Roth, H. Hergert, P. Papakonstantinou, T. Neff and H. Feldmeier, Phys. Rev. **C72** (2005) 034002.
  - [25] C. Forssen, P. Navratil, W.E. Ormand, E. Caurier, Phys. Rev. **C71** (2005) 044312.
  - [26] H. Feshbach, Ann. Phys. (N.Y.) **5** (1958) 357; **19** (1962) 287; C. Bloch and H. Feshbach, Ann. Phys. (N.Y.) **23** (1963) 47.
  - [27] W.C. Haxton and T. Luu, Nucl. Phys. **A690** (2001) 15; W.C. Haxton and T. Luu, Phys. Rev. Lett. **89** (2002) 182503.
  - [28] S.Y. Lee and K. Suzuki, Phys. Lett. **B91** (1980) 173; K. Suzuki and S.Y. Lee, Prog. Theor. Phys. **64** (1980) 2091.
  - [29] B.K. Jennings, Europhys. Lett. **72** (2005) 211.
  - [30] P.F. Bedaque and U. van Kolck, Ann. Rev. Nucl. Part. Sci. **52** (2002) 339.
  - [31] E. Braaten and H.-W. Hammer, cond-mat/0410417.
  - [32] C.A. Bertulani, H.W. Hammer and U. Van Kolck, Nucl. Phys. **A712** (2002) 37.
  - [33] A. Schwenk and C.J. Pethick, Phys. Rev. Lett. **95** (2005) 160401.
  - [34] P. Descouvemont, Phys. Rev. **C70** (2004) 065802.
  - [35] S.K. Bogner, A. Schwenk, R.J. Furnstahl and A. Nogga, Nucl. Phys. **A763** (2005) 59.
  - [36] P. Navratil, J.P. Vary and B.R. Barrett, Phys. Rev. **C62** (2000) 054311.
  - [37] M. Wloch, D.J. Dean, J.R. Gour, M. Hjorth-Jensen, K. Kowalski, T. Papenbrock and P. Piecuch, Phys. Rev. Lett. **94** (2005) 212501.
  - [38] B.K. Jennings, Europhys. Lett. **72** (2005) 216; Note that the Lee-Suzuki method [28] used in the No-Core Shell Model is identical to  $V_{\text{low } k}$  in the nuclear matter limit.
  - [39] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. **16** (1986) 1.
  - [40] M. Thies, Phys. Lett. **B162** (1985) 255; **B166** (1986) 23.
  - [41] E.D. Cooper and B.K. Jennings, Nucl. Phys. **A458** (1986) 717.
  - [42] B.K. Jennings and N. de Takacs, Phys. Lett. **B124** (1983) 302; G.E. Brown, B.K. Jennings and V.I. Rostokin, Phys. Rept. **50** (1979) 227; M. Ericson and T.E.O. Ericson, Ann. of Phys. **36** (1966) 323.
  - [43] E. Caurier, G. Martinez-Pinedo, F. Nowacki, A. Poves and A.P. Zuker, Rev. Mod. Phys. **77** (2005) 427.
  - [44] S.K. Bogner, T.T.S. Kuo, L. Coraggio, A. Covello and N. Itaco, Phys. Rev. **C65** (2002) 051301(R); A. Schwenk and A.P. Zuker, nucl-th/0501038.
  - [45] S. Goriely, M. Samyn, P.-H. Heenen, J. M. Pearson and F. Tondeur, Phys. Rev. **C66** (2002) 024326.
  - [46] M. Bender, P.-H. Heenen and P.-G. Reinhard, Rev. Mod. Phys. **75** (2003) 121.
  - [47] M.V. Stoitsov, J. Dobaczewski, W. Nazarewicz, S. Pittel and D.J. Dean, Phys. Rev. **C68** (2003) 054312; J. Dobaczewski and W. Nazarewicz, Prog. Theor. Phys. Suppl. **146** (2003) 70.
  - [48] P. Finelli, N. Kaiser, D. Vretenar and W. Weise, Nucl. Phys. **A735** (2004) 449.
  - [49] R.J. Furnstahl, J. Phys. **G31** (2005) S1357.
  - [50] A. Schwenk, G.E. Brown and B. Friman, Nucl. Phys. **A703** (2002) 745.
  - [51] R. Shankar, Rev. Mod. Phys. **66** (1994) 129.
  - [52] D. Zanchi and H.J. Schulz, Phys. Rev. **B61** (2000) 13609.
  - [53] C.J. Halboth and W. Metzner, Phys. Rev. Lett. **85** (2000) 5162; C.J. Halboth and W. Metzner, Phys. Rev. **B61** (2001) 7364.
  - [54] M. Salmhofer and C. Honerkamp, Prog. Theor. Phys. **105** (2001) 1.
  - [55] G. Murthy and S. Kais, Chem. Phys. Lett. **290** (1998) 199.
  - [56] G. Watanabe, K. Sato, K. Yasuoka and T. Ebisuzaki, Phys. Rev. **C66** (2002) 012801; Phys. Rev. **C68** (2003) 035806; Phys. Rev. **C69** (2004) 055805.
  - [57] C.J. Horowitz, M.A. Perez-Garcia and J. Piekarewicz, Phys. Rev. **C69** (2004) 045804; C.J. Horowitz, M.A. Perez-Garcia, D.K. Berry and J. Piekarewicz, Phys. Rev. **C72** (2005) 035801.
  - [58] C.J. Horowitz and A. Schwenk, nucl-th/0507033 and nucl-th/0507064.
  - [59] F. Sarazin *et al.*, Phys. Rev. **C70** (2004) 031302(R).
  - [60] K.M. O'Hara *et al.*, Science **298** (2002) 2179; J. Kinast *et al.*, Science **307** (2005) 1296.
  - [61] T. Bourdel *et al.*, Phys. Rev. Lett. **91** (2003) 020402; J. Zhang *et al.*, cond-mat/0410167.
  - [62] M. Greiner, C.A. Regal and D.S. Jin, Nature **426** (2003) 537 and cond-mat/0502539.
  - [63] C. Chin *et al.*, Science **305** (2004) 1128; M. Bartenstein *et al.*, cond-mat/0412712.
  - [64] S. Gupta *et al.*, Science **300** (2003) 1723; M.W. Zwierlein *et al.*, Nature **435** (2005) 1047.
  - [65] M.W. Zwierlein, A. Schirotzek, C.H. Schunck and W. Ketterle, cond-mat/0511197.